Active suppression of freeplay aeroelastic vibrations of ailerons by robust control methods with incomplete measurements

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Abstract

Purpose – The purpose of this paper is to analyze the active suppression of the nonlinear aeroelastic vibrations of ailerons caused by freeplay by robust H_∞ and linear quadratic Gauss (LQG) methods of control in case of incomplete measurements of the state of the system.

Design/methodology/approach – The flexible wing with nonlinear aileron with freeplay is treated as a plant-controller system with H_∞ and LQG controllers used to suppress the aeroelastic vibrations. The simulation approach was used for analyzing the impact of completeness of measurements on the efficiency and robustness of the controllers.

Findings – The analysis shows that the H_∞ method can be effectively used for suppression of nonlinear aeroelastic vibrations of aileron, although its efficiency depends essentially on completeness and types of measurements. The LQG method is less effective, but it is also able to prevent aileron vibrations by reducing their amplitudes to acceptable, safe level.

Research limitations/implications – Only numerical analysis was carried out for the problem described; thus, the proposed solution is of theoretical value at this stage of analysis, and its application to the real suppression of aeroelastic vibrations requires further research.

Practical implications – The work presents a potentially useful solution to the problem of interest and results are a theoretical basis for further research.

Social implications – This work may lead to a hot debate on the advantages and drawbacks of the active suppression of vibrations in the aeroelasticians community.

Originality/value – The work raises the important questions of practical stabilizability of the nonlinear aeroelastic systems, their dependence on completeness and types of measurements and robustness of the controllers.

Keywords Vibrations, Measurements, Robust Control, Aeroelasticity, Active suppression, Freeplay

Paper type Research paper

Introduction

Aeroelastic vibrations of aircraft structures, especially of lifting surfaces and controls, are very dangerous because they can cause disasters of aircraft, especially in case of mode-coupled bending-torsion-aileron flutter (Bisplinghoff *et al.*, 1955) or they are at least undesirable, such as nonlinear vibrations of controls, especially ailerons or elevators (Dowell, 2015). For over hundred years, the classic flutter was a reason of several disasters and malfunctions of aircraft of all types: from gliders, small general aviation aircraft, medium military training aircraft to heavy fast military aircraft. In a hundred-year history of aviation, a majority of aircraft, including the most famous ones, had greater or lesser problems with aeroelastic vibrations. Classical flutter of lifting surfaces and controls is very rapid; amplitude of vibrations grows quickly (exponentially) which usually causes damage of the structure of wing or controls that leads to disaster of aircraft. The classical flutter, however, is well known and thus is not so dangerous as it was in the past, which is confirmed by a relatively small number of disasters that happened in past 20 years. This is because of the reliability and accuracy of the methods of computing the critical velocity of flutter and the refined procedures of airborne flutter tests (Hodges and Pierce, 2011). According to the regulations, the critical speeds of any flutters have to exceed at least 15 per cent [big aeroplanes CS-25 (2016)] or 20 per cent [small and medium aeroplanes, CS-23 (2012)] the allowable speeds of aircraft. A quite different situation is in the case of other aeroelastic vibrations, especially vibrations of control surfaces, most commonly ailerons (Dowell, 2015). Such vibrations, caused usually by freeplay or hysteresis in aileron control systems, are usually nonlinear and occur at velocities much lower than those of critical flutter speeds and, what is even worse, lower than the operational speed limits (VD or VNE). An infamous example of such situation was a disaster of the F-117 stealth fighter in 1997 caused by nonlinear vibrations of aileron at the speed ~750 km/h, much lower than the critical flutter speed, ~1100 km/h. It thus should not be expected that

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the critical velocities of such vibrations will be higher than those of operational speeds of aircraft.

Nonlinear vibrations of controls occurred from time to time in the past: in the 1990s, the vibrations of elevators in some types of Airbus commercial aircraft were observed that resulted in extensive discussion between the manufacturer and government authority (Croft, 2001). Similarly, the aforementioned spectacular disaster of F-117 was caused by the nonlinear vibrations of aileron. Dowell (2015) stated that the occurrence of nonlinear flutter of limit cycle oscillations (LCO) type caused by freeplay in control surfaces was not documented in the public literature, although it was discussed in the aeroelastician community and was perhaps described in unpublished internal reports of aircraft companies and government authorities.

There is thus a need of explaining the physical mechanisms of such vibrations, that result in publication of a number of papers devoted to these problems, e.g. Kim and Lee (1996), Tang *et al.* (1998), Frampton and Clark (2000), Liu *et al.* (2002), Liu and Dowell (2005), Tang and Dowell (2010), Daochun *et al.* (2010), Xiang *et al.* (2014) and Kholodar (2016).

Nonlinear vibrations are most often the LCO with limited amplitude; thus, they need not be so destructive as divergent oscillations of classical flutter. There is even an opinion that the nonlinear vibrations of LCO type just prevents the occurrence of classical divergent flutter (Dowell, 2015). Nevertheless, such nonlinear vibrations are undesirable because they may result in material fatigue, upset of control systems or growing freeplays; thus, they should be prevented or suppressed.

Prevention of aeroelastic vibrations may be passive (assured in designing process and by the exploitation procedures) or active, that consists in damping of vibrations on their onset using automatic flight control system (AFCS) that has to be supplied by the additional system for measuring deflections of flexible aircraft structure. It should be mentioned here, that the active suppression need not damp the vibrations entirely; it is often sufficient to prevent the exponential increasing their amplitude keeping it on safe, low level.

The concept of the active suppression of vibrations has a long, over 40 years, history (Dowell, 2015). Though as early as in 1973, the active suppression of classical bending-torsional flutter of wing was successfully tested on the B-52 bomber aircraft (Roger *et al.*, 1975), an increasing of the critical flutter speed over 18 km/h (that is, 1.6 per cent of critical flutter speed) was rather symbolic. Later on, active suppression of classical linear mode-coupling type flutter had been practically abandoned, however, probably because it was considered to be too hazardous (e.g. disaster of the X-56A unmanned aircraft during testing active flutter suppression system, November 19, 2015, Warwick, 2015) and because of the fact that the regulations require the critical speeds of flutter phenomena be greater than the operational speed limits (CS-23, 2016, CS-25, 2012). The idea of active suppression of aeroelastic vibrations has been refreshed in the past decade in the context of suppression of aeroelastic vibrations that cannot be "shifted" beyond the operational speed limits, such as the flutter of very flexible wings and nonlinear vibrations of controls. The active suppression of nonlinear vibrations of controls of limit cycle oscillation type are very attractive, because these vibrations occur typically at the speeds much lower than those of critical flutter speed, and the existing FCS systems mounted routinely on the aircraft for flight control purposes can be used without extensive modifications for suppressing of such vibrations.

A significant progress in the area of active suppression of aeroelastic vibrations has been made recently, both theoretical and experimental (Block and Strganac, 1998; Ko *et al.*, 1998; Clark *et al.*, 2000; Frampton and Clark, 2000; Bialy *et al.*, 2014).

In this paper, the problem of active suppression of aeroelastic vibrations of aileron with freeplay in the stiffness nonlinearity is considered. In spite of restricted ability of measuring of the state of the system the robust H_∞ and linear quadratic Gauss (LQG) control method with output feedback has been proposed for active suppression of vibrations. The semirigid 3-degrees of freedom linear structural model of wing with nonlinear aileron coupled with aerodynamic potential model of incompressible flow was used for aeroelastic analysis (Bisplinghoff *et al.*, 1955). It will be shown that nonlinear vibrations can be effectively damped by using the AFCS systems mounted on aircraft assuming that measurements are complete enough.

The new achievements concern assessing the impact of incomplete measurements on efficiency and ability of active suppression of vibrations caused by freeplay. Although incomplete measurements were considered (Lee and Singh 2007), according to best knowledge of the present author, no deeper analysis of this problem is presented in the literature.

The structure of the paper is as follows. In Section 1, the nonlinear aeroelastic model of wing, its form used in control theory and its linearized version are presented. Next, the general concept of active suppression of vibration will be presented together with the methods of control linear quadratic regulator (LQR) and H_∞ used for this purpose, generalized for nonlinear problems (Cimen, 2012). Later on, the problem of incomplete measurements and their impact on robustness and efficiency of both methods will be described. In Section 2, the results of simulations of active suppression of freeplay vibrations by both H_∞ and LQG methods are presented and comparison of both methods in this context is given. Next, the analysis of impact of incomplete measurements on efficiency and robustness of H_∞ method used for suppression of ailerons vibration is presented.

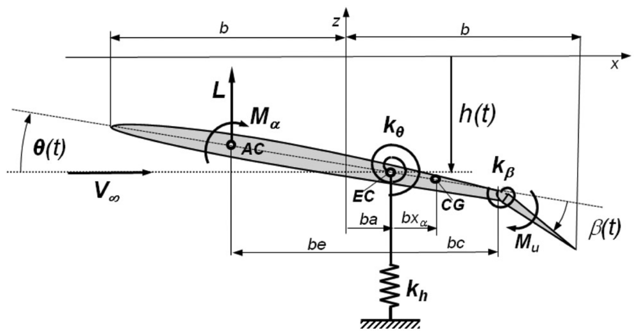
Aeroelastic model of wing and aileron with freeplay

The classical semi-rigid model of flexible wing with aileron having freeplay nonlinearity and incompressible aerodynamics has been used for analysis of active suppression of vibrations (Figure 1) (Bisplinghoff *et al.*, 1955; Tewari, 2015). It has been assumed that wing and aileron can perform small vibrations around their equilibrium positions, angles of attack of wing and aileron are also assumed to be small and resulting aerodynamic forces are linear functions of them; thus, the linear, quasi-steady aerodynamic model can be used (Bisplinghoff *et al.*, 1955). The aeroelastic model of wing and aileron has then the form:

$$(M_S - M_A(V_\infty))\,\ddot{q} + (D_S - D_A(V_\infty))\,\dot{q} + (K_S - K_A(V_\infty))\,q = f_u(u) \qquad (1)$$

where the state vector $q(t) = [h(t),\ \theta(t),\ \beta(t)]^T$ describes bending $h(t)$ and torsion $\theta(t)$ of wing and aileron's deflection

Figure 1 Semi-rigid model of flexible wing and aileron with freeplay nonlinearity



$\beta(t)$, M_S, D_S and K_S are, respectively, the mass, damping and stiffness matrices of the wing, $M_A(V_\infty)$, $D_A(V_\infty)$ and $K_A(V_\infty)$ are, respectively, the aerodynamics mass, damping and stiffness matrices of the wing that depend on undisturbed velocity of flow V_∞, $f_u(u) = [0,\ 0,\ M_u(u)]^T$ is the vector of external excitations in which $M_u(u)$ is the control moment applied to the aileron that depends on control u served by the control system.

The aileron freeplay is modeled by the nonlinear moment $M_u(u;\delta)$ that depends on applied control u (Figure 2)

$$M_u(u;\delta) = \begin{cases} 0 & |u| \le \delta \\ c_{mu}(u - sign(u)\,\delta) & |u| > \delta \end{cases} \quad (2)$$

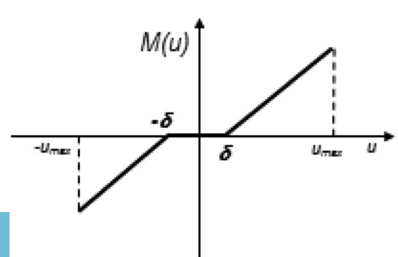
where c_{mu} is the coefficient of the control moment and δ is the width of the freeplay zone. This is a middle nonlinearity, because it can be linearized locally.

The aeroelastic model of wing and aileron equations (1) and (2) can be written in the standard form of the system of ordinary differential equations of the first order:

$$\dot{x} = f(x,u) = \left[\dot{q}, -(M_S - M_A)^{-1}\left((D_S - D_A)\,\dot{q} + (K_S - K_A)\,q - f_u(u)\right)\right]^T \quad (3)$$

where $x(t) = [q(t), \dot{q}(t)]^T \in R^n$ is the state vector of the standard model dimension of $n = 6$, $u(t) \in R^m$ is the vector of control dimension of $m = 1$, whereas $f(\cdot;\cdot)$: $R^n \times R^m \to R^n$ is the nonlinear operator of the dynamics of the aeroelastic system.

Figure 2 Model of aileron freeplay



Active suppression of vibrations as the control problem

The concept of the active suppression of vibrations consists in using the automatic control system with feedback loop with controller K (Figure 3), that is used for damping of the aeroelastic vibrations by getting them asymptotically to the equilibrium $x(t) \to 0$. In the considered problem of damping the nonlinear aeroelastic vibrations of aileron, it is important to prevent their unbounded exponential growth and to assure small amplitude of the limit cycle. Thus, instead of requiring the full damping of vibrations, one may postulate a weaker requirement, consists in limiting the amplitude of vibrations to some small, technically acceptable level, $\|x(t)\| \le \Theta$, where Θ is the maximal acceptable amplitude of vibrations.

The dynamical model of active suppression of vibration of the aeroelastic system given by equation (3) supplied by the observation model and control model can be written in the general nonlinear form used in control theory (Cimen, 2012) (Figure 4):

$$\dot{x} = f(x, u; \Delta) + w_x \quad (4)$$

$$y = c(x; \Delta) + w_y \quad (5)$$

$$u = k(y) \quad (6)$$

where $y \in R^p$, $p \le n$, is the output (measurements) vector, $c(\cdot)$: $R^n \to R^p$ is the nonlinear output operator, $k(\cdot)$: $R^n \to R^m$ is the nonlinear operator of feedback control (controller), $w_x(t) \in R^n$ i $w_y(t) \in R^p$ are the exogenous external disturbances of model dynamics and measurements (which are not considered in this work), whereas the vector Δ stands for internal disturbances that describe unknown properties ("lack of knowledge") of the physical system.

The optimal control methods are defined basically for linear systems, but they can be extended to wide class of nonlinear systems as well (Cimen, 2012). In this approach, the nonlinear model in equations (4) to (6) is linearized in the vicinity of the actual state of the model:

Figure 3 Model of active suppression of wing-aileron vibrations

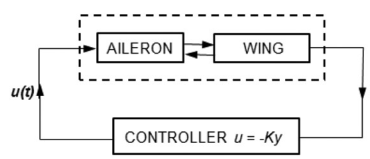
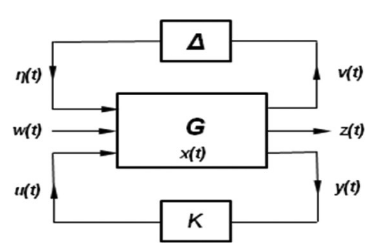


Figure 4 Control theory representation of active suppression model



$$\dot{x} = A(x;\Delta)x + B(x;\Delta)\,u + w_x \tag{7}$$

$$y = C(x;\Delta)\,x + w_y \tag{8}$$

$$u = -K(x)\,y \tag{9}$$

where the matrices of dynamics $A \in R^{n\times n}$, control $B \in R^{n\times m}$, measurements $C \in R^{p\times n}$ and the gain matrix (controller) $K \in R^{m\times n}$ are the following Jacobians of the nonlinear operators *f*, *c* and *k*:

$$A(x) = \frac{\partial f(x,u)}{\partial x}, \quad B(x) = \frac{\partial f(x,u)}{\partial u},$$
$$C(x) = \frac{\partial c(x)}{\partial x}, \quad K(x) = -\frac{\partial k(y)}{\partial y} \tag{10}$$

The linearized model in equations (7) to (9) will be used for active suppression of aeroelastic vibrations of aileron.

The controller K may be the classical regulator or it can be designed based on the optimal or robust control theory (Zhou and Doyle, 1999; Lewis *et al.*, 2012; Lavretsky and Wise, 2013; Skogestad and Postlewhite, 2005). Optimal controller should assure the minimization of the error vector $z(t) \in R^r$ defined by the regulated error equation:

$$z = C_1\,x + D_{12}\,u \tag{11}$$

where $C_1 \in R^{r\times n}$, $D_{12} \in R^{r\times n}$ are the state and control errors matrices. It can be attained by minimization either of the H_∞ norm:

$$\|G(s)\|_\infty := \max_{\|w(t)\|_2=1} \|z(t)\|_2 = \max_{w(t)\neq 0} \frac{\|z(t)\|_2}{\|w(t)\|_2} \tag{12}$$

or the performance index:

$$\begin{aligned} \mathcal{J} &= \|z^T z\|_2^2 \\ &= \|\left(x^T C_1^T C_1\,x + 2x^T C_1^T D_{12}\,u + u^T D_{12}^T D_{12}\,u\right)\|_2^2 \end{aligned} \tag{13}$$

A special case of equation (13) is the performance index used in the LQR and LQG optimal control methods that uses the state and control errors:

$$\mathcal{J}(x,u) = \int_0^\infty \left(x^T Q x + u^T R u\right) dt \tag{14}$$

where $Q = C_1^T C_1 \in R^{n\times n}$ and $R = D_{12}^T D_{12} \in R^{m\times m}$ are some appropriate weighting matrices. Controllers K that minimize the performance criteria in equations (12) to (14) have the general form:

$$K = R^{-1} B^T P \tag{15}$$

where $P \in R^{n\times n}$, $P = P^T > 0$ is the matrix being the solution of the stationary Riccati equation:

$$0 = A^T P + PA - PBR^{-1}B^T P + Q \tag{16}$$

Problem of incomplete state measurements

The basic method of optimal control is the LQR method. Its advantages are high efficiency, high phase and gain margins that guarantee its good robustness. The main disadvantage of the LQR method is the necessity of measuring of the full state vector, $y = x$, which is difficult in practice. In the considered case of active suppression of vibrations, only the aileron's deflection and its rate measurements are easily available because they are measured for the AFCS system of aircraft control. Measurements of the deflections of the aircraft structure is difficult because it requires installation an additional measuring equipment on aircraft, that can use accelerometers, laser or cameras (Tewari, 2015) which is cumbersome in practice. Restricted ability of measuring of the full state vector precludes, in principle, the use of the LQR method for active suppression of vibrations. The solution to this problem is to use other method of optimal control, such as the LQG or H_∞ method, that allows incomplete measurements, in which the number of measured variables may be smaller than the dimension of the state vector:

$$p = \dim(y) \leq n \tag{17}$$

In such a case, the controllers K of both methods, LQG and H_∞, have to use some estimation vector $\hat{x}(t) \in R^n$ of the state vector $x(t)$:

$$u = -K\hat{x} \tag{18}$$

Estimation vector $\hat{x}$ is the solution of linear observer equation (e.g. the Kalman filter) having the form (Zhou and Doyle, 1999; Lewis *et al.*, 2012; Lavretsky and Wise, 2013; Skogestad and Postlewhite, 2005):

$$\dot{\hat{x}} = A\hat{x} + Bu + L(C\hat{x} - y) \tag{19}$$

where $L \in R^{n\times p}$ is the measurements injection matrix.

In the present work, it will be shown that not only the number of measurements p but also their types (deflections or velocities) have an essential influence on efficiency of suppression of aileron vibrations. This concerns both LQG and H_∞ methods.

Evaluation of robustness

The H_∞ method serves in addition an opportunity to evaluate the impact of various factors on the robustness of the suppression of vibrations. In particular, the influence of incomplete measurements may be assessed this way.

The robustness of the H_∞ method can be evaluated based on the concept of internal disturbances Δ introduced in the model equations (8) to (10) by the so called coprime factorization of the transmittance of the system (Zhou and Doyle, 1999; Skogestad and Postlewhite, 2005). The state space representation of the transmittance of the aeroelastic model:

$$G_\Delta = \begin{bmatrix} A(\Delta) & B(\Delta) \\ C(\Delta) & 0 \end{bmatrix} \tag{20}$$

can be factored in the form of (left) coprime factorization:

$$G_\Delta = (M + \Delta_M)^{-1}(N + \Delta_N) \tag{21}$$

where M and N are factors of the transmittance G_Δ for $\Delta = 0$, and Δ_M, Δ_N are the transmittances of disturbances Δ.

If these factored disturbances are bounded in the H_∞ norm by $\varepsilon_{max} > 0$:

$$\|[\Delta_M \, \Delta_N]\|_\infty < \varepsilon_{max} \tag{22}$$

and if the controller K_∞ is stable and fulfils the condition:

$$\| \begin{bmatrix} K_\infty \\ I \end{bmatrix} (I + GK_\infty)^{-1} M^{-1} \|_\infty \le \frac{1}{\varepsilon_{max}} \tag{23}$$

then this controller K_∞ will be robust and the robustness margin ε_{max} is equal to:

$$\varepsilon_{max} = \sqrt{1 - \| [N \, M] \|_H^2} \tag{24}$$

As the coprime factors N and M depend on the matrices A, B and C of the physical model (Skogestad and Postlewhite, 2005), the robustness margin ε_{max} enables one to evaluate the robustness of the K_∞ controller for the given system. In particular, one is able to check quantitatively the influence of incompleteness of measurements, defined by the measurement matrix C of the model, on the robustness of the K_∞ controller and thus its ability to suppress the aeroelastic vibration for different number and types of measurements. There is no such option for LQG method.

Description of LQG and H_∞ methods

The general form of the controller $u = Ky$ has the form (Zhou and Doyle, 1999; Skogestad and Postlewhite, 2005):

$$\begin{bmatrix} \dot{\hat{x}} \\ u \end{bmatrix} = K \begin{bmatrix} \hat{x} \\ y \end{bmatrix} \tag{25}$$

where $\hat{x}(t)$ is (hidden) estimate of the state $x(t)$ and K is the generalized controller. Specific controllers for LQG and H_∞ methods are defined as follows.

LQG controller

Controller of the LQG method has the form:

$$K_{LQG} = \begin{bmatrix} A - BR^{-1}B^TX - YC^TV^{-1}C & YC^TV^{-1} \\ -R^{-1}B^TX & 0 \end{bmatrix} \tag{26}$$

where matrices X and Y are solutions of the Riccati equations:

$$A^TX + XA - XBR^{-1}B^TX + Q = 0 \tag{27}$$

$$YA^T + AY - YC^TV^{-1}CY + W = 0 \tag{28}$$

Matrices V and W are either the power spectral density matrices for disturbances of model dynamics and measurements or simply they are design matrices that have to be tuned (Skogestad and Postlewhite, 2005). The observer equation has the form:

$$\dot{\hat{x}}(t) = A\hat{x} + Bu + YC^TV^{-1}(C\hat{x} - y) \tag{29}$$

H_∞ controller

The H_∞ method is suboptimal, the parameter of suboptimality γ has to fulfill the condition:

$$\gamma > \gamma_{min} = 1/\varepsilon_{max} \tag{30}$$

where γ_{min} is the (minimal) optimal value that depends on the specific problem via ε_{max} defined by equation (24).

Controller of the H_∞ method has the form (Skogestad and Postlewhite, 2005):

$$K_\infty(\gamma) = \begin{bmatrix} A - BB^TX + \gamma^2\left((1-\gamma^2)I + XZ\right)^{-T}ZC^TC & \gamma^2\left((1-\gamma^2)I + XZ\right)^{-T}ZC^T \\ B^TX & 0 \end{bmatrix} \tag{31}$$

where matrices X and Z are solutions of the Riccati equations:

$$A^TX + XA - XBB^TX + C^TC = 0 \tag{32}$$

$$ZA^T + AZ - ZC^TCZ + BB^T = 0 \tag{33}$$

The optimal parameter γ_{min} is given by:

$$\gamma_{min} = \sqrt{1 + \rho(XZ)} \tag{34}$$

where $\rho(A)$ denotes the spectral radius of the matrix A. The observer equation has the form:

$$\dot{\hat{x}}(t) = A\hat{x} + Bu + Z\left((1-\gamma^2)I + XZ\right)(C\hat{x} - y) \tag{35}$$

In the limit $\gamma \to \infty$ the H_∞ method tends to the LQG method (Zhou and Doyle, 1999), which might suggest that the H_∞ method is generally better than LQG method. It is, however, by no means obvious for linearized problems.

The controllers of the LQG and H_∞ methods defined by the equations (26) to (35) will be used for active suppression of aeroelastic vibrations of aileron.

Simulation analysis of active suppression of aileron vibrations

The developed model of active suppression of nonlinear aeroelastic vibration of aileron with freeplay has been used for analysis the impact of incomplete measurements on the efficiency and robustness of the compared methods H_∞ and LQG. The simulation (time-marching) approach was used for numerical analysis because of the nonlinearity of the model.

The analysis concerns:

- checking the efficiency of active suppression of aileron vibrations;
- comparison of both methods H_∞ and LQG; and
- checking the impact of incomplete measurements on efficiency and robustness of active suppression.

The analysis was carried out for the following cases:

- full state measurements, $h(t)$, $\theta(t)$, $\beta(t)$, $\dot{h}(t)$, $\dot{\theta}(t)$, $\dot{\beta}(t)$ $n_y = 6$;
- measuring of three velocities, $\dot{h}(t)$, $\dot{\theta}(t)$, $\dot{\beta}(t)$, $n_y = 3$;
- measuring of three deflections, $h(t)$, $\theta(t)$, $\beta(t)$, $n_y = 3$;
- measuring of individual variable: $h(t)$ and $\beta(t)$, $n_y = 1$;
- measuring the rate of aileron deflection only $\dot{\beta}(t)$, $n_y = 1$;
- measuring of deflection and rate of aileron, $\beta(t)$, $\dot{\beta}(t)$, $n_y = 2$; and
- the best minimal combination of measurements: torsional angular velocity $\dot{\theta}(t)$ and rate of aileron $\dot{\beta}(t)$, $n_y = 2$.

Comparison of H_∞ and LQG methods was done for all these cases.

The following plots will show time histories (in seconds) of aileron's deflection $\beta(t)$ (deg), wing torsion angle $\theta(t)$ (deg) and wing bending $h(t)$ (m). Only the deflections $h(t)$, $\theta(t)$ and $\beta(t)$ are shown, velocities $\dot{h}(t)$, $\dot{\theta}(t)$, $\dot{\beta}(t)$ behave similarly, so the plots will be omitted for savings.

The model data are (Figure 1) wing area $S = 10.3$ m^2, wing semichord $b = 0.345$ m, aerodynamic center position $a = -0.50$, elastic axis position $e = -0.25$, aileron's leading edge position $c = 0.652$, wing mass $m = 71$ kg, wing moment of inertia $I_\theta = 27.0$ kgm^2, aileron moment of inertia $I_\beta = 0.88$ kgm^2, bending natural frequency $f_h = 5.1$ Hz, torsional natural frequency $f_\theta = 7.7$ Hz, aileron natural frequency $f_\beta = 10.0$ Hz, aileron deflection limit $\beta_{max} = 30°$, aileron freeplay $\delta = 0.2$ (20 per cent β_{max}), aileron control moment coefficient $c_{mu} = 200$ Nm and lift curve slope $dc_L/d\alpha = 5.54$.

Level flight with velocity $V_\infty = 72.0$ m/s at altitude $H = 2{,}000$ m was assumed for which the air density is $\rho = 1.0065$ kg/m^3.

Classical bending-torsional-aileron flutter

For verification of the reliability of the basic aeroelastic model, the classical bending-torsional-aileron flutter has been simulated first. There is no freeplay in aileron and the active suppression is switched off. One obtains typical exponentially divergent vibrations (Figure 5).

The critical flutter speed $V_{kr} = 65.9$ m/s is of 10 per cent lower than that of bending-torsional flutter, $V_{kr} = 71.6$ m/s, which is consistent with the classical model of flutter (Bisplinghoff *et al.*, 1955). The model seems thus to be reliable and appropriate for testing the active suppression of nonlinear aileron vibrations (Figure 6).

Full state measurements, ny = nx = 6

The H_∞ method suppress vibrations very efficiently, which arises from high robustness margin, $\varepsilon_{\max} = 0.44$ (according to Skogestad and Postlewhite (2005); a good robustness margin is considered to be $\varepsilon_{\max} = 0.25$); the LQG method is not able to suppress the vibrations; LCO of significant amplitudes occur, although there is no catastrophic growth of vibrations, and bending-torsion-aileron flutter is prevented.

Incomplete measurements, velocities, ny = 3

There is no decreasing of efficiency of both H_∞ and LQG method in comparing with full measurements case. For H_∞ method, it is confirmed by robustness margin $\varepsilon_{\max} = 0.42$ that is still very good. The LQG method behaves similarly to the full measurements case, it is still able to prevent bending-torsion-aileron flutter (Figure 7).

Incomplete measurements, deflections, ny = 3

Very surprisingly, neither H_∞ nor LQG methods are able to suppress vibrations, despite that robustness margin $\varepsilon_{\max} = 0.291$ is still very good. There is essential decreasing in efficiency of H_∞ method in comparing with the same number of velocities measured. Vibrations grow quickly and after 0.4 s the aileron deflection attains the limiting value 30°. The LQG method is much more robust in this case, although it is neither able to suppress vibrations or stabilize them in a safe limit cycle (Figure 8).

Failures in suppressing vibrations both of LQG and H_∞ methods with displacement-type measurements was quite surprising, because stable LCO was expected, as in the case of velocity measurements, described above. This means that not only the number but also the type of measurements plays a crucial role in robustness of the controllers. It is also visible, that the robustness margin defined in H_∞ method is not necessarily an adequate measure of its effectiveness, at least in the context of active suppression of vibrations.

Incomplete measurements, deflection of wing, ny = 1

Neither H_∞ nor LQG methods are able to suppress vibrations entirely, limit cycles appears, but both methods prevent bending-torsion-aileron flutter. The LQG method is more efficient in this case. The robustness margin $\varepsilon_{\max} = 0.016$ is very small and this might explain worse effectiveness of H_∞ method comparing with the LQG method in this case (Figure 9).

Incomplete measurements, deflection of aileron, ny = 1

Despite that only one variable is measured here, there is close similarity to the third case with measurements of all three deflections $\beta(t)$, $h(t)$, $\theta(t)$. Neither H_∞ nor LQG methods are able to suppress vibrations, which is now reflected in the

Figure 5 Classical bending-torsional-aileron flutter. Supercritical aileron's vibrations β(t)

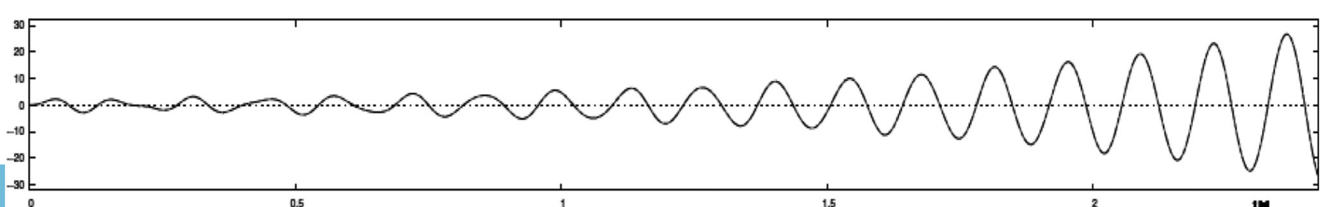
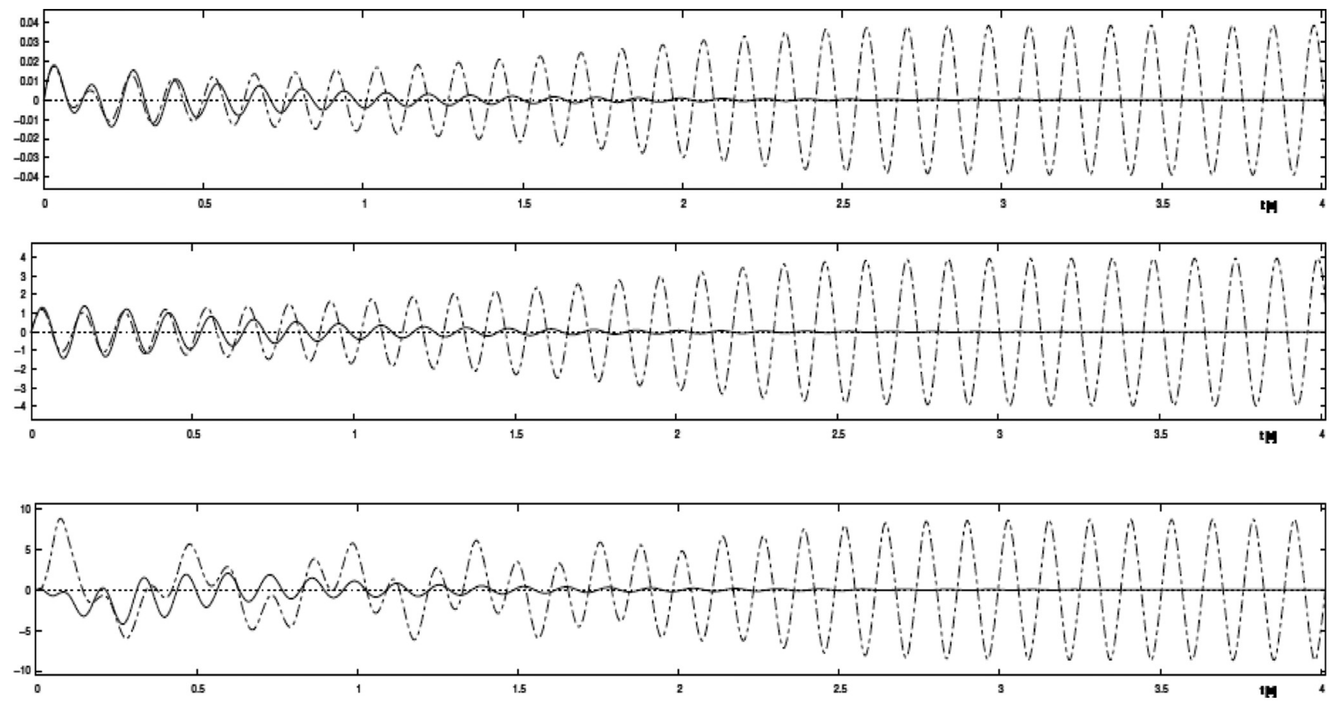
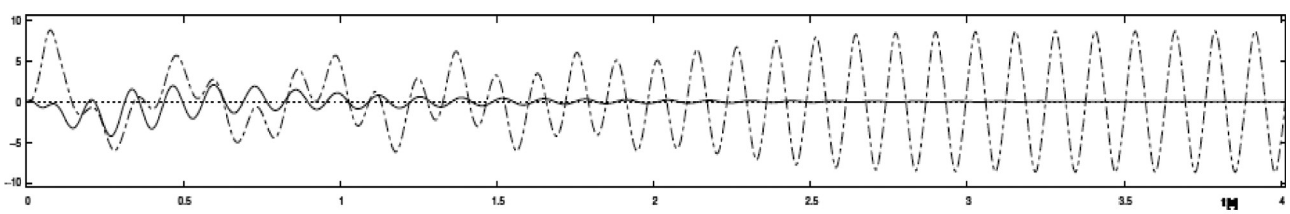


Figure 6 Vibrations of *h(t)*, *θ(t)* and *β(t)*



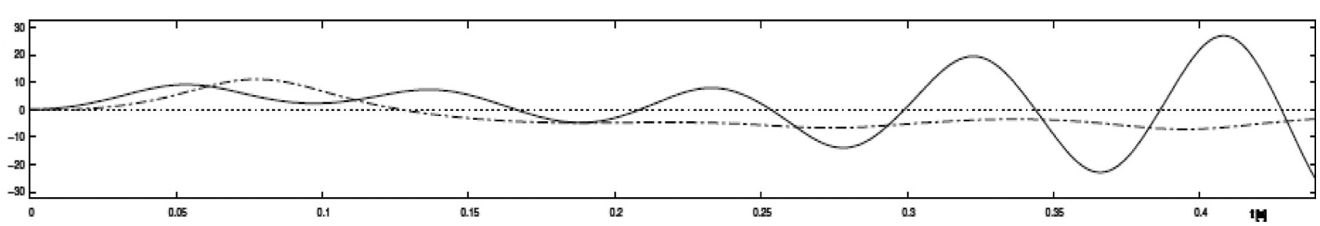
Notes: Full measurements, $n_y = 6$. (- - - - LQG —— H_∞)

Figure 7 Vibrations of *β(t)*



Notes: Incomplete measurements, $\dot{h}(t)$, $\dot{\theta}(t)$, $\dot{\beta}(t)$, $n_y = 3$. (- - - - LQG —— H_∞)

Figure 8 Vibrations of *β(t)*



Notes: Incomplete measurements, $\beta(t)$, $h(t)$, $\theta(t)$, $n_y = 3$. (- - - - LQG —— H_∞)

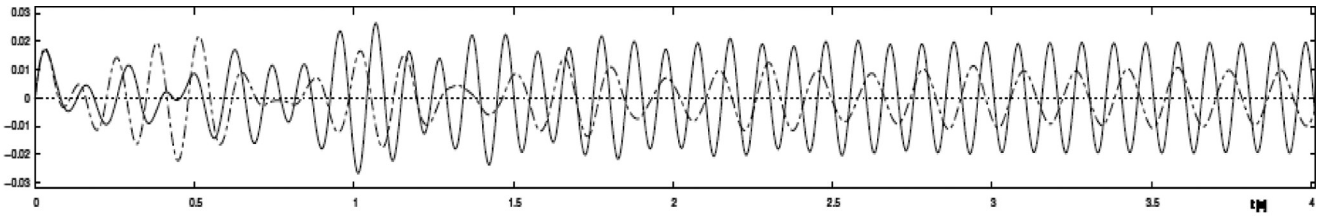
robustness margin $\varepsilon_{\max} = 0.0394$ that is much lower than in the aforementioned third case. It is then obvious that not the number but the type of measured variable is crucial for efficiency and robustness of the active suppression of vibrations (Figure 10).

Incomplete measurements, rate of aileron deflection, ny = 1

This is the most interesting case, as aileron's deflection rate $\dot{\beta}(t)$ is easily available measurement (Figure 11).

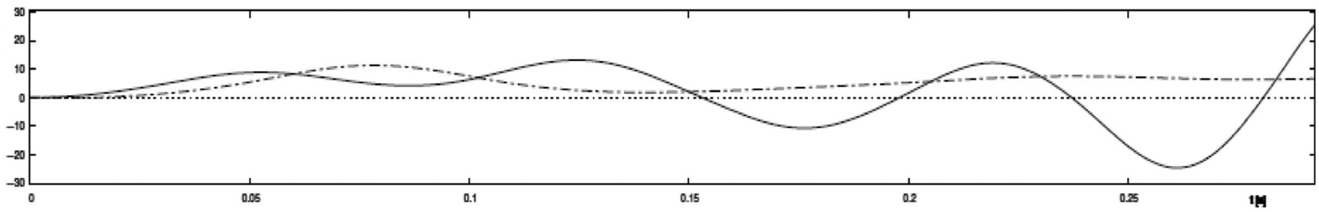
Results are somewhat unexpected, as despite the fact that only one variable is measured, the suppression of vibrations is

Figure 9 Vibrations of *h(t)*



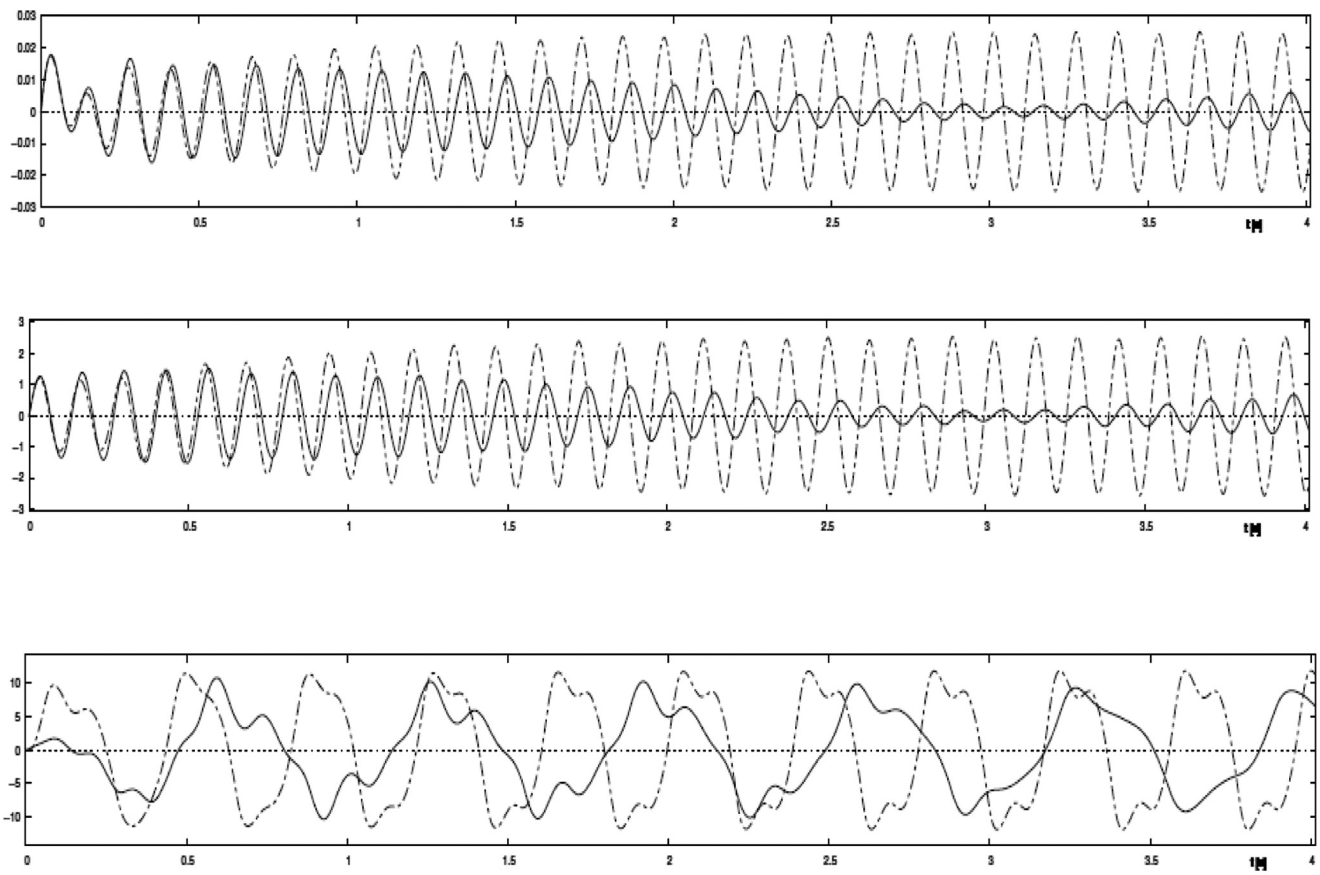
Notes: Incomplete measurements, $h(t)$, $n_y = 1$. (- - - - LQG —— H_∞)

Figure 10 Vibrations of *β(t)*



Notes: Incomplete measurements, $\beta(t)$, $n_y = 1$. (- - - - LQG —— H_∞)

Figure 11 Vibrations of *h(t), θ(t), β(t)*



Notes: Incomplete measurements, $\beta(t)$, $n_y = 1$. (- - - - LQG —— H_∞)

quite satisfactory. Although neither H_∞ nor LQG methods are able to suppress vibrations entirely, the limit cycles appear and bending-torsion-aileron flutter is prevented. The H_∞ method is much more efficient than LQG method in this case, which is reflected in quite good robustness margin, $\varepsilon_{\max} = 0.1775$, It is somewhat surprising that, despite the rate of aileron's deflection is measured, the vibrations of aileron are quite large, whereas suppression of bending and torsional vibrations of wing is much more effective.

Incomplete measurements, deflection and rate of aileron, ny = 2
Unfortunately, adding measurements of aileron's deflection $\beta(t)$ to the measurements of its rate $\dot{\beta}(t)$ does not improve the suppression of vibrations comparing to measurement of rate $\dot{\beta}(t)$ alone. The results are almost the same, which is confirmed by the same robustness margin, $\varepsilon_{\max} = 0.1775$ (Figure 12).

The best minimal combination of measurements: torsional angular velocity $\dot{\theta}(t)$ and rate of aileron $\dot{\beta}(t)$, ny = 2
The conclusion that can be drawn from all the cases analyzed previously is that the best result that can be obtained by the H_∞ method in active suppression of aileron vibrations in the sense of full suppression of them with the use of a minimal set of measured variables can be attained by one of two combinations of measurements: $\dot{\beta}(t)$ with either $\dot{\theta}(t)$ or $\dot{h}(t)$ (Figures 13 and 14).

Similar to the case of full measurements, the H_∞ method suppress vibrations very effectively, which is confirmed by good robustness margins, $\varepsilon_{\max} = 0.26$ and $\varepsilon_{\max} = 0.3049$, respectively, that are comparable to that of full measurements ($\varepsilon_{\max} = 0.44$). The LQG method is not able to suppress the vibrations, LCO of significant amplitudes occur; moreover, the amplitudes of aileron deflections are high and because of freeplay, less regular than those of bending and torsion vibrations of wing.

It is visible that the best minimal combination of measurements is that of $\dot{\beta}(t)$ and $\dot{\theta}(t)$. Measuring of $\dot{h}(t)$ provides worse results, although not by much.

Finally, it is important to state that the dependence of the efficiency of active suppression of aeroelastic vibrations upon the types of measurements is not affected by the features of both LQG and H_∞ methods. In particular, it does not depend on the weighting matrices Q and R of the LQG method.

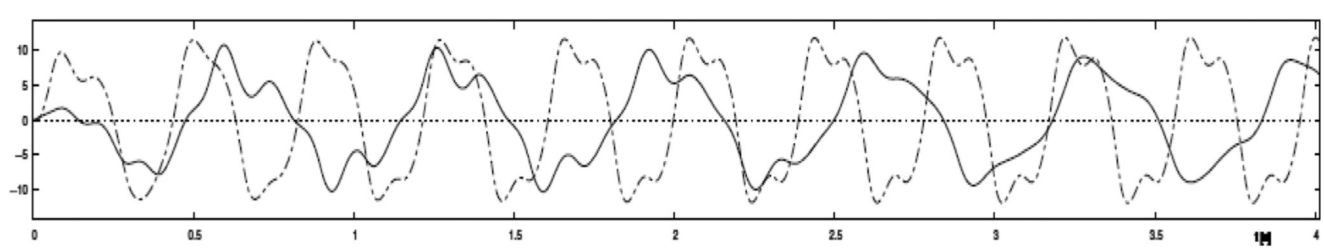
Conclusion

The simulation numerical analysis of the active suppression of the aeroelastic vibrations of nonlinear aeroelastic system of flexible wing with freeplay nonlinearity in aileron has shown that such nonlinear vibrations can be suppressed by H_∞ robust method of control quite effectively. The H_∞ method is able to damp the vibrations much quicker than the LQG method in most cases, but the ability of both methods to suppress the vibrations depends essentially on the completeness of measurements of the state variables: when they are complete (full state is available) the H_∞ method is very effective because of high level of robustness achieved in this case. Moreover, it has been also shown that incomplete measurements may also be sufficient for effective damping of vibrations; however, the efficiency of suppression depends not only on the number but also on the type of measurements (displacements or velocities). In the extreme case of measuring, only one state component, the robustness of the method, is not sufficient to damp the vibrations at all. However, the use of both H_∞ and LQG methods enables one to prevent the occurrence of flutter by reducing the amplitudes of the limit cycles of such vibrations to acceptable, safe level, that in practice may give the pilot enough time to perform rescue action. It also turned out that the full measurements are not necessary for effective suppression of vibrations. Even the minimal combination of measurements consists of torsion rate and aileron's deflection rate provides suppression that is almost as effective as that obtained with full measurements.

The use of the robustness margin defined in H_∞ method as a measure of efficiency of active suppression has been confirmed only partially. In most cases, it provided good qualitative information on the efficiency of the suppression, but there were cases when the margins were low or high and the suppressions were, adversely, high or low. This means that the robustness margin is not entirely adequate measure of effectiveness of active suppression of vibrations, and that the other factors are important, which requires further research.

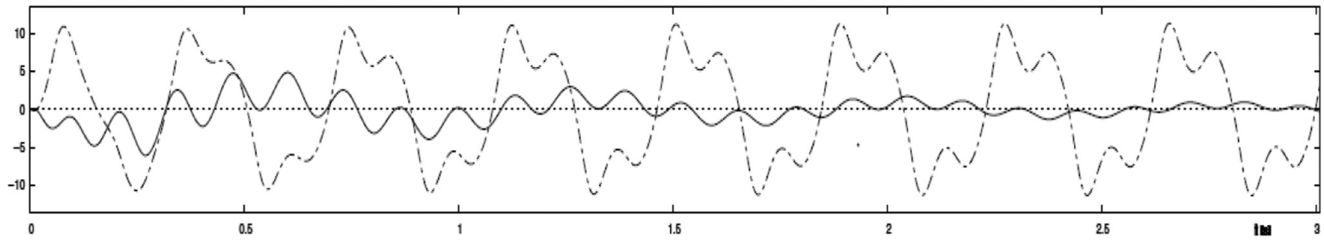
To summarize, the ability of both H_∞ and LQG methods to suppress the vibrations of mildly nonlinear aeroelastic systems, such as the aileron with freeplay, has been confirmed in the present work. The still open question is whether they will be efficient in suppression of vibrations of aeroelastic systems with hard nonlinearities, such as of hysteresis type.

Figure 12 Vibrations of $\beta(t)$



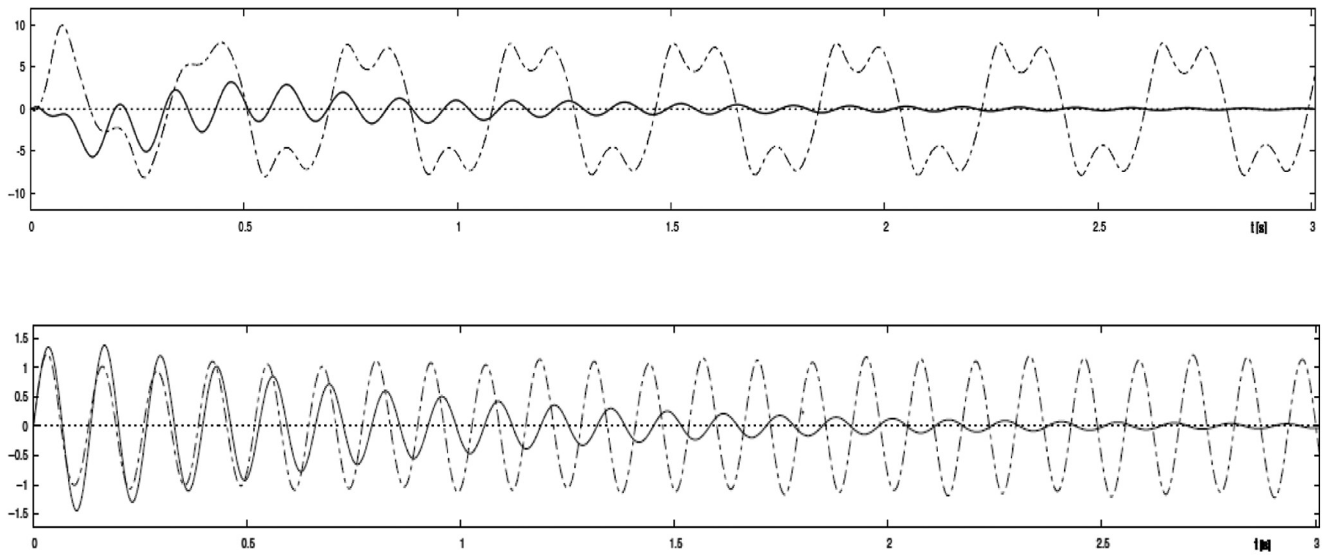
Notes: Incomplete measurements, $\beta(t)$, $\dot{\beta}(t)$, $n_y = 2$. (- - - - LQG —— H_∞)

Figure 13 Vibrations of $\beta(t)$



Notes: Incomplete measurements, $\dot{\beta}(t)$, $\dot{h}(t)$, $n_y = 2$. (- - - - LQG —— H_∞)

Figure 14 Vibrations of $\beta(t)$ and $\theta(t)$



Notes: Incomplete measurements, $\dot{\beta}(t)$, $\dot{h}(t)$, $n_y = 2$. (- - - - LQG —— H_∞)

Further work

Further works may concern on answering the question why the measurements of velocities are better than those of deflections for active suppression of vibrations. Also, the problem of adequate measure of efficiency of the H_∞ method is open. The other area of investigation can be the active suppression of vibrations of highly nonlinear aeroelastic systems, such as those with hysteresis nonlinearities.

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